2017 4/28 at Chiba

Asymptotically Background Free Quantum Gravity and Its Cosmological Implications

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Renormalizable Asymptotically Background Free Quantum Gravity

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BRST Conformal Symmetry

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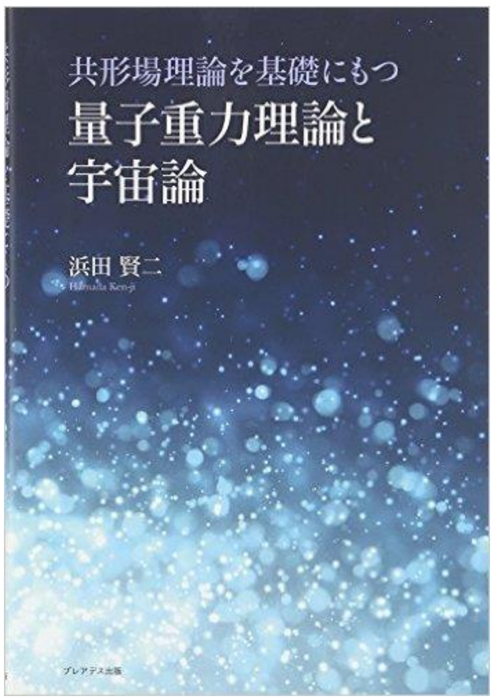
Book

「共形場理論を基礎にもつ量子重力理論と宇宙論」
"Quantum Gravity and Cosmology based on Conformal Field Theory"
(Pleiades Publishing, 2016)



The contents of today's talk are almost all written in this book

Introduction

The goal of quantum gravity is to understand phenomena beyond the Planck scale

Limit of Einstein Gravity (1)

Problems of quantizing Einstein action

- The coupling const. (Newton const.) has dimension

  ➔ perturbatively non-renormalizable

- There exists a solution with spacetime singularity

  ➔ when performing path integral over the metric field,
  we cannot exclude such a singular configuration
  because Einstein action is finite for such a configuration

- The Einstein action is not bounded below (like φ^3 -theory)

  ➔ unstable even when quantizing it non-perturbatively

Limit of Einstein Gravity (2)

Einstein gravity is unitary within the perturbation theory

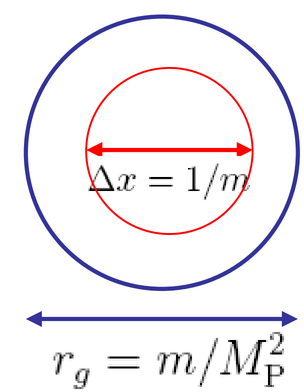
- No ghost modes
- Physical mode is given by graviton propagating on a <u>fixed</u> background

But, unitarity problem ocures in a strong gravity region

Elementary excitation with Planck mass ➔ Black Hole,

For $m > M_{\mathrm{P}}$

because Compton wave length $<$ Schwarzschild radius



Information of particle is confined inside of horizon and lost
➔ point-like particle picture breaks down

Einstein gravity cannot go beyond the Planck scale

↑ play a role of UV cutoff

Several Approaches Beyond Einstein Gravity

- Attempts to find a <u>manifestly finite</u> theory that has no divergences or can eliminate divergences using symmetries and equations of motion

  Ex. supergravity, superstring

  - In general, it is <u>not</u> defined in 4 dimensions, and defined in a <u>perturbative</u> way about Einstein theory
  - It gives a <u>local</u> effective theory, and so there is no scale other than Planck mass (➔ UV cutoff, after all)

- Attempts to find a <u>renormalizable</u> theory in 4 dimensions by introducing 4-derivative gravitational actions

  - It has UV divergences, but they are renormalizable
  - It gives a <u>non-local</u> effective theory

Features of 4th-Order Quantum Gravity

4th order quantum gravity can resolve the problems existing in Einstein gravity as follows

- The coupling constant becomes dimensionless
- The action given by the square of curvature tensor becomes bounded from below

But, in general, ghost modes appear in a perturbative treatment

4th order propagator: $$\frac{1}{m^2p^2+p^4} \to \frac{1}{p^2} - \frac{1}{p^2+m^2}$$

ghost mode

The models are classified by how to challenge the unitarity problem

- Lee-Wick-Tomboulis approach
- Horava approach
- BRST CFT (asymptotically background free) approach

- Lee-Wick-Tomboulis approach (in1970s) :

  Consider resummed propagator

  for asymptotically free 4th order theories with $\beta = -\beta_0 g^3$

$$\frac{1}{m^2 p^2 + \beta_0 p^4 \log(p^2/\Lambda^2)} = \frac{1}{p^2 [m^2 + \beta_0 p^2 \log(p^2/\Lambda^2)]}$$

➔ Real ghost pole disappears ➔ complex pole
(This idea is still effective at IR, but no good at UV)

- Horava approach :

  Give up Lorentz sym. ➔ make ghosts non-dynamical

- BRST CFT approach :

  Partially use a non-perturbative method
  BRST conformal inv. arises as a part of diff. inv. in UV limit
  ➔ make ghosts unphysical!

Asymptotically Background Free Quantum Gravity

Here, gives brief summary of the basic structure

Later, present the formulation using dimensional regularization

Basic Assumption

From the cosmological observations, it has been found that the spectrum of early universe is scale-invariant, or conformally invariant

So, assume that all dimensionless couplings should be conformally invariant

(Mass parameter is OK, because it can be neglected in UV limit)

Gravitational actions are then given by the two combinations

$$\begin{cases} C^2_{\mu\nu\lambda\sigma} = R^2_{\mu\nu\lambda\sigma} - 2R^2_{\mu\nu} + \frac{1}{3}R^2 & \text{square of Weyl tensor} \\ G_4 = R^2_{\mu\nu\lambda\sigma} - 4R^2_{\mu\nu} + R^2 & \text{Euler density (=Gauss-Bonnet)} \end{cases}$$

Renormalizable ABF Quantum Gravity

$\mathrm{sgn.} = (-1, 1, 1, 1)$

The QG Action (Weyl + Euler + Lower derivatives)

$$I = \int d^4x \sqrt{-g} \left[\underbrace{-\frac{1}{t^2} C^2_{\mu\nu\lambda\sigma} - bG_4}_{\text{conformally invariant (no } R^2\text{)}} + \frac{M^2}{2} R - \Lambda + \underset{\substack{\uparrow \\ \text{conformal matter}}}{\mathcal{L}_M} \right]$$

Weyl action is positive-definite, and includes Riemann curvature tensor

Thus, a spacetime configuration that Riemann curvature tensor diverges such as Schwarzschild BH is excluded from path integral because the action diverges for such a singular configuration

➔ No spacetime singularities

For the moment, consider energy scale beyond Planck mass and neglect the Einstein term and the cosmological term (🡐 considered later)

Perturbation about Conf. Flat Spacetime

"*t*" is a unique dimensionless gravitational coupling in the theory, which has negative beta function (asymptotic freedom)

On the other hand, since Euler term does not have kinetic term at tree level, "*b*" is not an independent coupling, which is expanded by *t*

At high energy, $t \longrightarrow 0 \quad C_{\mu\nu\lambda\sigma} \longrightarrow 0$ (conformally flat)

singularities are removed in the UV limit

Perturbation theory is defined about $C_{\mu\nu\lambda\sigma} = 0$ as

$$g_{\mu\nu} = e^{2\phi} \hat{g}_{\mu\lambda} (\delta^{\lambda}{}_{\nu} + th^{\lambda}{}_{\nu} + \cdots), \quad tr(h) = 0$$

Conformal-factor (exactly)

BRST conformal symmetry

Traceless tensor field (perturbatively)

This dynamics is ruled by Weyl action

Conformal-Factor Dynamics (Key Point)

The action *I* does not have the kinetic term of conformal-factor field
Dynamics of conformal-factor is <u>induced</u> from the measure

$$Z = \int [dg\cdots]_{\underline{\underline{g}}} \exp(iI)$$

$$g_{\mu\nu} = e^{2\phi}\bar{g}_{\mu\nu} = e^{2\phi}(\hat{g}e^{th})_{\mu\nu}$$

$$= \int [d\phi dh\cdots]_{\underline{\underline{\hat{g}}}} \exp(\underline{\underline{iS(\phi)}} + iI)$$

(# physical quantities)

Practical measure defined on the background

Jacobian to ensure diffeomorphism inv.
= Wess-Zumino actions for conformal anomalies: $\phi^n F_{\mu\nu}^2,\ \phi^n \bar{C}_{\mu\nu\lambda\sigma}^2,\ \phi^{n+1}\bar{\Delta}_4\phi,\ \cdots$

Kinetic term is induced at the lowest order independent of *t* :

$$S_{\rm R} = -\frac{b_1}{(4\pi)^2}\int d^4x\sqrt{-\hat{g}}\left[2\phi\hat{\Delta}_4\phi + \left(\hat{G}_4 - \frac{2}{3}\hat{\nabla}^2\hat{R}\right)\phi\right] \qquad b_1 > 0$$

(positive-definite)

Riegert-Wess-Zumino action

4th-order conformally invariant op.

$$b_1 = \frac{1}{360}(N_S + 11N_F + 62N_A) + \frac{769}{180}$$

c.f. Liouville action in 2DQG

Diffeomorphism Invariance

$$\delta_\xi g_{\mu\nu} = g_{\mu\lambda}\nabla_\nu \xi^\lambda + g_{\nu\lambda}\nabla_\mu \xi^\lambda$$

ξ^μ : gauge parameter

Metric field is now expanded as

$$g_{\mu\nu} = \underset{=}{e^{2\phi}} \bar{g}_{\mu\nu} \qquad \bar{g}_{\mu\nu} = (\hat{g}\, e^{th})_{\mu\nu} = \hat{g}_{\mu\lambda}\left(\delta^\lambda{}_\nu + t h^\lambda{}_\nu + \frac{t^2}{2}(h^2)^\lambda{}_\nu + \cdots\right)$$

Conformal factor
<u>Exactly</u>

Traceless tensor field
<u>Perturbatively</u>

gauge-fixed later

Diffeomorphism is then decomposed as

$$\delta_\xi \phi = \xi^\lambda \partial_\lambda \phi + \frac{1}{4}\hat{\nabla}_\lambda \xi^\lambda$$

$$\delta_\xi h_{\mu\nu} = \frac{1}{t}\left(\hat{\nabla}_\mu \xi_\nu + \hat{\nabla}_\nu \xi_\mu - \frac{1}{2}\hat{g}_{\mu\nu}\hat{\nabla}_\lambda \xi^\lambda\right) + \xi^\lambda \hat{\nabla}_\lambda h_{\mu\nu} + \frac{1}{2} h_{\mu\lambda}\left(\hat{\nabla}_\nu \xi^\lambda - \hat{\nabla}^\lambda \xi_\nu\right)$$
$$+ \frac{1}{2} h_{\nu\lambda}\left(\hat{\nabla}_\mu \xi^\lambda - \hat{\nabla}^\lambda \xi_\mu\right) + o(t)$$

two modes completely decoupled!

BRST Conf. Inv. as a Part of Diff. Inv.

Consider gauge parameter satisfying conformal Killing vectors

$$\hat{\nabla}_\mu \zeta_\nu + \hat{\nabla}_\nu \zeta_\mu - \frac{1}{2}\hat{g}_{\mu\nu}\hat{\nabla}_\lambda \zeta^\lambda = 0$$

Gauge transformations with ζ^μ at t = 0 (UV limit) become

characteristic of diff.

$$\delta_\zeta \phi = \zeta^\lambda \partial_\lambda \phi + \underline{\frac{1}{4}\hat{\nabla}_\lambda \zeta^\lambda}$$

dimensionless scalar with shift term

$$\delta_\zeta h_{\mu\nu} = \zeta^\lambda \hat{\nabla}_\lambda h_{\mu\nu} + \frac{1}{2} h_{\mu\lambda}\left(\hat{\nabla}_\nu \zeta^\lambda - \hat{\nabla}^\lambda \zeta_\nu\right) + \frac{1}{2} h_{\nu\lambda}\left(\hat{\nabla}_\mu \zeta^\lambda - \hat{\nabla}^\lambda \zeta_\mu\right)$$

dimensionless tensor

Changing ζ^μ with ghost c^μ, we obtain BRST conformal symmetry

This gauge symmetry is so strong because RHS is field-dependent!

(# Radiation gauge: we can gauge-fix traceless tensor fields properly such that gauge d.o.f. reduce to conformal Killing vectors only)

BRST Conformal Symmetry

Background-metric independence can be represented in terms of BRST conformal symmetry

↖ arises as a part of diff. symmetry

➔ represents a gauge equivalency among all theories connected to one another by conformal transformations

$$ds^2 \cong \Omega^2 ds^2$$

This symmetry makes ghost modes unphysical!

Owing to this property, we can choose any conformally flat background without changing any physics

Therefore, we can formulate quantum gravity theory as a conventional quantum field theory defined on Minkowski background

➔ This is a great advantage of when quantizing gravity

(because we can use conventional methods of quantum field theory)

Asymptotic Background Freedom

BRST CFT + perturbations (by single "*t*")

BRST conformal symmetry mixes positive- and negative-metric modes
➔ Ghosts are not gauge invariant

Non-perturbative theory given by treating conformal-factor exactly

It describes totally-fluctuated quantum spacetime, and thus there is no graviton picture propagating in a fixed spacetime

cf. Asymptotically free quantum gravity in 1970's

Free + perturbations (by two couplings)

Gauge symmetry in free part does not mix gravitational modes at all
➔ non-unitary (ghosts become gauge inv.)

perturbative (all modes are treated in perturbation)

based on graviton picture

Renormalizable ABF Quantum Gravity Using Dimensional Regularization

First, I will briefly discuss gravitational counterterms and conformal anomalies in curved space

Then, using its results, I will formulate renormalizable ABF quantum gravity

Dimensional Regularization

Advantages:

- It preserves gauge symmetries, including diffeomorphism invariance
- It is the only regularization method we can carry out higher loop calculations

Significant property:

In exactly 4 dimensional space, measure contributions such as conformal anomalies come from divergent quantity $\delta^4(0) = \langle x|x'\rangle|_{x'\to x}$

evaluated using DeWitt-Schwinger method

In dim. reg., however, it is regularized to zero as $\delta^D(0) = \int d^D k = 0$

➔ Path integral results are independent of how to choose the measure, and measure contributions (conformal anomalies) are contained between *D* and 4 dimensions ➔*D*-dep. of action is quite important !

$\frac{1}{D-4}$ x $D-4$ ➔ finite (= conformal anomalies)

from loop in action

Fixing *D*-dependence of Gravitational Actions

K.H., Phys. Rev. D89 (2014) 104063

From RG analysis of correlation functions among EM tensor by Hathrell, it has been shown that gravitational counterterms for QED and QCD in curved space can be unified into 2 forms at all orders

Euclidean sgn.

$$S_g = \int d^D x \sqrt{g} \{ a_0 C^2_{\mu\nu\lambda\sigma} + \overbrace{b_0 G_D}^{b_0 G_4 + c_0 H^2} \}$$

Bare couplings b_0 and c_0 are related through RG equations

where

$$G_D = G_4 + (D-4)\chi(D)H^2 \qquad H = \frac{R}{D-1}$$

← ambiguities fixed! (conf. anomaly fixed)

$\chi(D)$ is a finite function of *D* only that can be determined order by order

First three terms are explicitly calculated as $\chi(D) = \frac{1}{2} + \frac{3}{4}(D-4) + \frac{1}{3}(D-4)^2 + \cdots$

In general, for conformal couplings, only these two counterterms are necessary, and also in renormalizable ABF QG

Renormalizable ABF Quantum Gravity

Quantum gravity action ($a_0 \to 1/t_0^2$)

conformal matters

$$S = \int d^D x \sqrt{g} \left[\frac{1}{t_0^2} C_{\mu\nu\lambda\sigma}^2 + b_0 G_D - \frac{M_0^2}{2} R + \Lambda_0 + \mathcal{L}_M \right]$$

Perturbation is carried out about a conformally flat spacetime

$$g_{\mu\nu} = \underline{\underline{e^{2\phi}}} \bar{g}_{\mu\nu} \qquad \bar{g}_{\mu\nu} = (e^{t_0 h_0})_{\mu\nu} = \delta_{\mu\nu} + t_0 h_{0\mu\nu} + \frac{t_0^2}{2} h_{0\mu}^{\lambda} h_{0\lambda\nu} + \cdots$$

treated exactly
in exponential form

Significant feature of renormalization

$$Z_\phi = 1$$

Conformal-factor field is not renormalized from requirement of diffeomorphism invariance, because there is no coupling constant for this field

Laurent expansion of b_0

1-loop correction given before (coupling indep.)

$$b_0 = \frac{\mu^{D-4}}{(4\pi)^{D/2}} \sum_{n=1}^{\infty} \frac{b_n}{(D-4)^n}$$

Pure pole and $b_1 = b + b_1'(t)$

Since Euler term does not have a kinetic term at tree level, the coupling for this term should be removed and residues b_n are expanded by t

Euler term is then expanded as

Kinetic term (=Riegert action)

$$\begin{aligned} & b_0 \int d^D x \sqrt{g} G_D \\ &= \frac{\mu^{D-4}}{(4\pi)^{D/2}} \int d^D x \Big\{ \Big(\frac{b_1}{D-4} + \frac{b_2}{(D-4)^2} + \cdots \Big) \bar{G}_4 \\ &\quad + \Big(b_1 + \frac{b_2}{D-4} + \cdots \Big) \Big(2\phi \bar{\Delta}_4 \phi + \bar{E}_4 \phi + \frac{1}{18} \bar{R}^2 \Big) \\ &\quad + [(D-4) b_1 + \cdots] \Big(\phi^2 \bar{\Delta}_4 \phi + \frac{1}{2} \bar{E}_4 \phi^2 + \cdots \Big) + \cdots \Big\} \end{aligned}$$

← counterterms

← new WZ actions + new counterterms

Dynamics of conformal-factor field are induced quantum mecahnically

Propagator of conformal-factor field $\propto \frac{\mu^{4-D}}{b} \frac{1}{k^4}$ (expanded by 1/b)

Weyl term is expanded as (also in gauge-field part, $C^2_{\mu\nu\lambda\sigma} \to F^2_{\mu\nu}$)

$$\frac{1}{t_0^2}\int d^D x \sqrt{g} C^2_{\mu\nu\lambda\sigma} = \frac{1}{t_0^2}\int d^D x e^{(D-4)\phi} \bar{C}^2_{\mu\nu\lambda\sigma}$$

$$= \int d^D x \left\{ \frac{1}{t_0^2} \bar{C}^2_{\mu\nu\lambda\sigma} + \frac{D-4}{t_0^2} \underline{\phi} \bar{C}^2_{\mu\nu\lambda\sigma} + \cdots \right\}$$

$$t_0 = \mu^{2-D/2} Z_t t$$

$$h_{0\mu\nu} = Z_h h_{\mu\nu}$$

Kinetic term (=*1/k*[4] : gauge fixed) and self-interactions of tensor field

WZ action = induced interactions

Beta function is calculated at one-loop level as

$$\beta_t \equiv \mu \frac{dt}{d\mu} = -\beta_0 t^3 \qquad \beta_0 = \frac{1}{(4\pi)^2}\left\{ \frac{1}{240}(N_S + 6N_F + 12N_A) + \frac{197}{60} \right\}$$

Now, this indicates <u>asymptotic background freedom</u>

because conformal-factor field still fluctuates non-perturbatively in UV region

Diffeomorphism Inv. Effective Action

WZ action of conformal anomaly is physical quantity to preserve diff. inv.

- Weyl part (and running coupling)

$$\Gamma_{\mathrm{W}} = \left[\frac{1}{t^2} - 2\beta_0 \underline{\phi} + \beta_0 \log\left(\frac{k^2}{\mu^2} \right) \right] \sqrt{-\bar{g}} \bar{C}^2_{\mu\nu\lambda\sigma}$$

$$= \frac{1}{\bar{t}^2(p)} \sqrt{-g} C^2_{\mu\nu\lambda\sigma}$$

For simplicity, ϕ is here taken to be a constant

↑

k = momentum on the background (= comoving mom. in cosmology)

where

$$\bar{t}^2(p) = \frac{1}{\beta_0 \log(p^2/\Lambda^2_{\mathrm{QG}})} \qquad \Lambda_{\mathrm{QG}} = \mu e^{-\frac{1}{2\beta_0 t^2}}$$

new physical scale ($\mu \frac{d\Lambda_{\mathrm{QG}}}{d\mu} = 0$)

↑

Physical momentum: $\underline{p^2 = \frac{k^2}{a^2}}$ with $a = e^{\phi}$ ← diff. inv. combination on the full metric

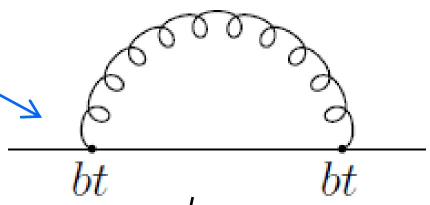
- Riegert part

$$b_1 = b\left[1 - a_1 \bar{t}^2(p) + \cdots\right]$$

coefficient receives corrections

Explicit Demonstrations of Non-Renormalization Theorem ($Z_\phi = 1$)

WZ interaction



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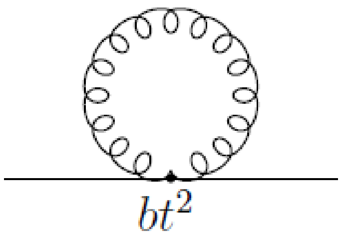


= UV finite

$$\frac{2b}{(4\pi)^2}k^4\left[-3\frac{t^2}{(4\pi)^2}\left(\frac{1}{\bar{\epsilon}}-\log\frac{z^2}{\mu^2}+\frac{7}{6}\right)\right] \qquad \frac{2b}{(4\pi)^2}k^4\left[3\frac{t^2}{(4\pi)^2}\left(\frac{1}{\bar{\epsilon}}-\log\frac{z^2}{\mu^2}+\frac{7}{12}\right)\right]$$

in Feynman gauge

Technical comments:

z: infinitesimal fictitious mass (IR regularization)

$1/\bar{\epsilon} = 1/\epsilon - \gamma + \log 4\pi$

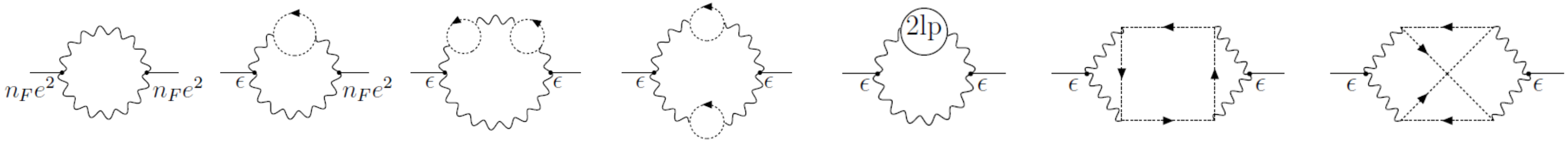
Propagator: $1/k^4 \to 1/(k^2+z^2)^2$

$D = 4 - 2\epsilon$

This mass is not gauge invariant ➔ cancel out !

Remark : Einstein action <u>cannot</u> be considered as the mass term due to the existence of exponential conformal-factor

Two-point function at e^4 in ABF QG coupled to QED

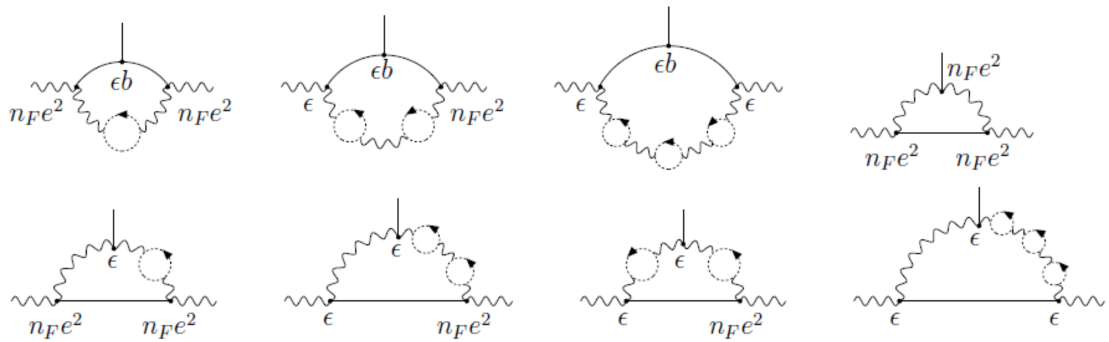


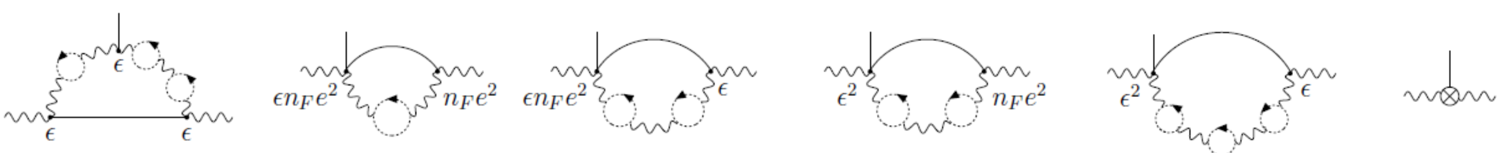
= finite

(also checked at e^6)

Vertex function ($\phi F_{\mu\nu}^2$) at e^6

$$\epsilon = (4 - D)/2$$





= finite

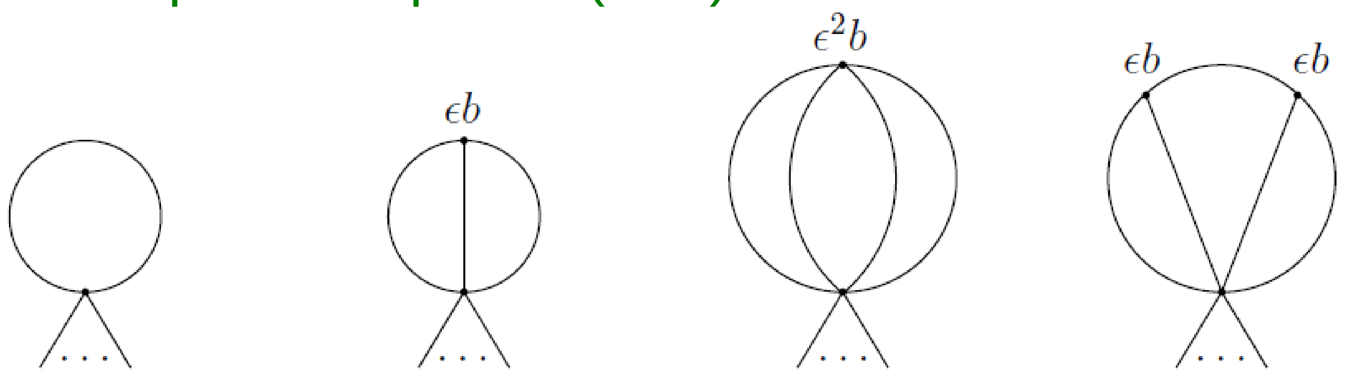
These diagrams are renormalized by the $Z_\phi = 1$ condition

K.H., PTP 108 (2002) 399

Anomalous Dimension of Cosmological Constant

Corrections up to 3 loops of o(1/b^3)

$\epsilon = (4-D)/2$



Anomalous dimension

$$\gamma_\Lambda = -\frac{\mu}{\Lambda}\frac{d\Lambda}{d\mu} = \frac{4}{b} + \frac{8}{b^2} + \frac{20}{b^3}$$

b = coeff. of Riegert action

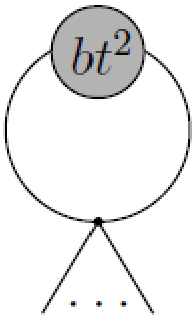
Propagator = *1/b*

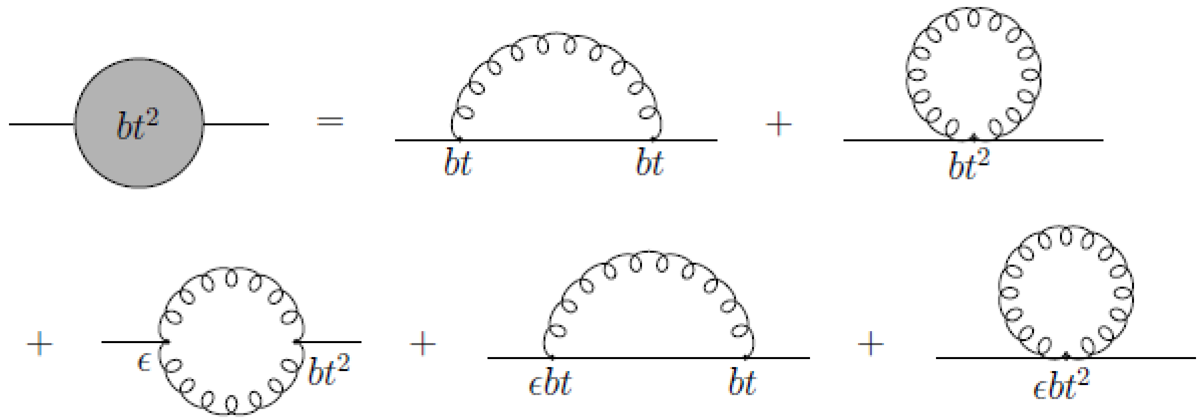
The result agrees with exact solution derived from BRST conformal symmetry algebraically:

$$\gamma_\Lambda = 2b\left(1-\sqrt{1-\frac{4}{b}}\right) - 4$$

Two-Loop Anomalous Dimensions of $O(t^2/b)$

in Landau gauge





finite $\quad Z_\phi = 1$

Anomalous dimension

$$\gamma_\Lambda\big|_{o(\alpha_t)} = -\frac{310}{9}\frac{1}{b}\frac{\alpha_t}{4\pi} \qquad \alpha_t = \frac{t^2}{4\pi}$$

K.H. and M. Matsuda, PR D93(2016)064051

Gravitational Physical Quantities

Physical scales *A* should be RG invariant such that $\mu \frac{d}{d\mu} A = 0$

- Dynamical IR scale:

$$\Lambda_{\mathrm{QG}} = \mu e^{-\frac{1}{2\beta_0 t^2}} \qquad\qquad \beta_t = -\beta_0 t^3$$

- Physical cosmological constant = effective potential (=observed value):

$$\Gamma|_V = \int d^4x V(\phi) = v \int d^4x e^{4\phi}$$

Since $Z_\phi = 1$, effective potential becomes RG invariant: $\mu \frac{dv}{d\mu} = 0$

$$\begin{aligned} v \;=\; & \Lambda + \frac{\Lambda}{b}(7 - 2\log 4\pi) - \frac{9\pi^2 M^4}{2b^2}\left(\frac{25}{3} - 4\log 4\pi\right) \\ & - \left(\frac{\Lambda}{b} - \frac{9\pi^2 M^4}{2b^2}\right)\log\frac{64\pi^2\Lambda}{b\mu^4} - \frac{6\pi M^2}{b}\sqrt{\frac{\Lambda}{b} - \frac{9\pi^2 M^4}{4b^2}}\arccos\left(\frac{3\pi M^2}{2\sqrt{b\Lambda}}\right) \\ & + \frac{5}{128}\alpha_t^2 M^4\left(\log\frac{(4\pi)^2\alpha_t^2 M^4}{16\mu^4} - \frac{21}{5}\right) \end{aligned}$$

K.H. and M. Matsuda, arXiv:1704.03962

- Physical Planck mass (🡸 effective action)

(# S-matrix is not defined because there is no free particle states in ABF QG)

Application to Cosmology

Why we can see quantum gravity phenomena

Naïve Questions for Universe

- Why universe is expanding

  Einstein's gravitational force is always <u>attractive</u>.
  Nevertheless, why universe expands against such a force.

  → need repulsive force in very early universe

- Why early fluctuations are so small

  Friedmann solution is <u>unstable</u>. So, universe gradually deviates from it.
  Nevertheless, universe even now can be almost described by it.

  Thus, in order that our universe continues more than 10 billion years, initial deviations must be very small, as observed by WMAP.

  On the other hand, since early universe be in a melting pot of high energy reactions, fluctuations (=deviations) seem so large naively

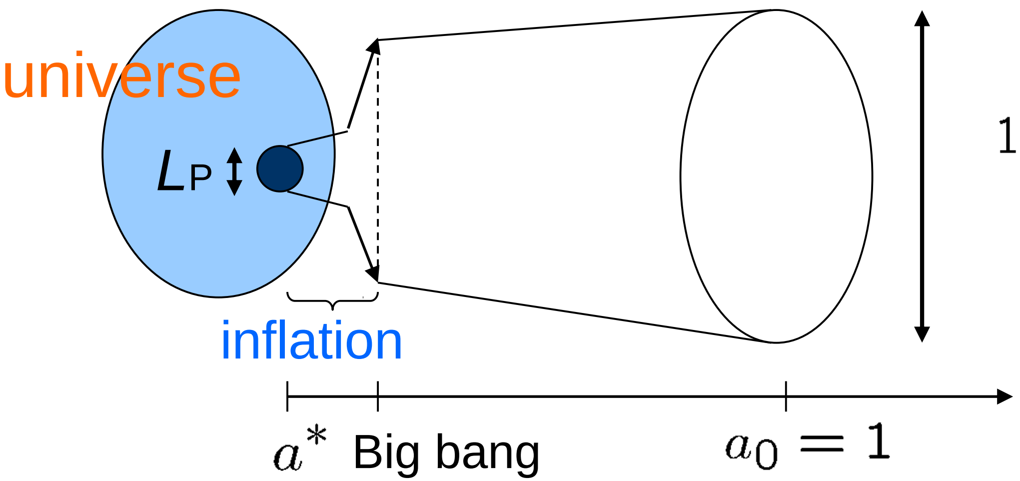
  → need novel mechanism to make fluctuations small

⇨ To solve these questions, the idea of inflation was proposed

Why Quantum Gravity

The typical inflationary scenario requires that the universe grows up about 10^{60} times in order to explain the flatness problem and so on

Trans-Plankian problem: $L_{\rm P} = 1/H_0 \times a^* \Leftarrow \quad a^* \simeq 10^{-60}$



$$
\begin{aligned}
1/H_0 &= 2998/h \text{ Mpc } (h = 0.72) \\
&= 1.3 \times 10^{28} \text{ cm}
\end{aligned}
$$

(= size of universe we can observe today)

a (scale factor)

Initial conditions of universe would be given by quantum gravity

How to Break the Wall of Planck Scale

I proposed the model with

Background-metric independence
= BRST conformal invariance

because it implies there is no fixed scale and no special point

The concept of time and distance is lost
in the UV limit

This also implies that we abandon a point-particle picture
propagating in a fixed spacetime

$\Downarrow$

We can then break the wall of Planck scale

A Consequence from Scale Invariance

Of course, the present universe is not scale invariant
It indicates that there is a spacetime transition in very early epoch
and so there is a scale separating two phases

quantum spacetime
(scale invariant)

classical spacetime
(with a scale)

Novel dynamical scale $\Lambda_{\rm QG}$

The existence of this scale is indicated from
asymptotically free behavior of the coupling *t*

Correlation length of QG is given by $\xi_\Lambda = 1/\Lambda_{\rm QG}$

This is "minimal length" we can measure

Quantum Gravity Inflation

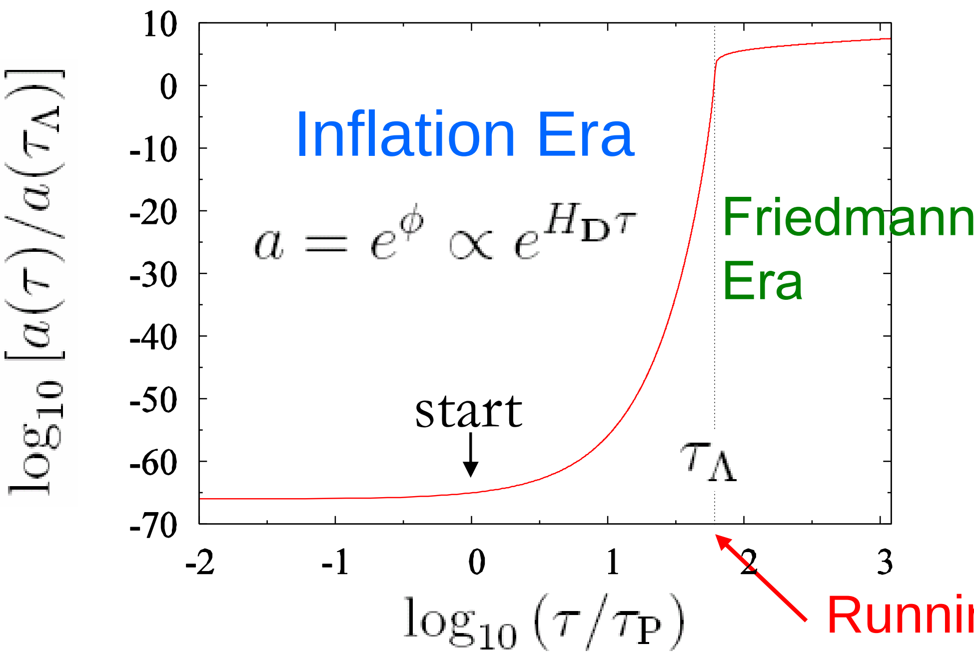
Evolution of the early universe can be described as a violating process of conformal invariance

Inflationary Solution

Equation of Motion with dynamical factor *B*

$$-\frac{b_1}{8\pi^2}B(\tau)\partial_\eta^4\phi + 3M_P^2 e^{2\phi}(\partial_\eta^2\phi + \partial_\eta\phi\partial_\eta\phi) = 0$$

rewritten in physical time: $d\tau = a d\eta$



Dynamical factor (modeling)

$$B(\tau) = 1 - a_1\bar{t}^2(\tau) + \cdots = \frac{1}{1 + a_1\bar{t}^2(\tau)}$$

Time-dep. running coupling

$$\bar{t}^2(\tau) = \frac{1}{\beta_0 \log(1/\tau^2\Lambda_{QG}^2)}$$

Inflation starts at Planck time

$$\tau_P = 1/H_D \quad H_D = M_P\sqrt{\frac{8\pi^2}{b_1}}$$

End at dynamical time

$$\tau_\Lambda = 1/\Lambda_{QG}$$

Running coupling diverges and thus
B → 0 (conformal dynamics disappears)

Evolution of Universe

$\xi_\Lambda = 1/\Lambda_{QG} \ (\gg l_{pl})$

Number of e-foldings

$$\mathcal{N}_e = \log \frac{a(\tau_\Lambda)}{a(\tau_P)} \simeq \frac{H_D}{\Lambda_{QG}}$$

➔ $\boxed{\Lambda_{QG} \simeq 10^{17} \ \mathrm{GeV}}$

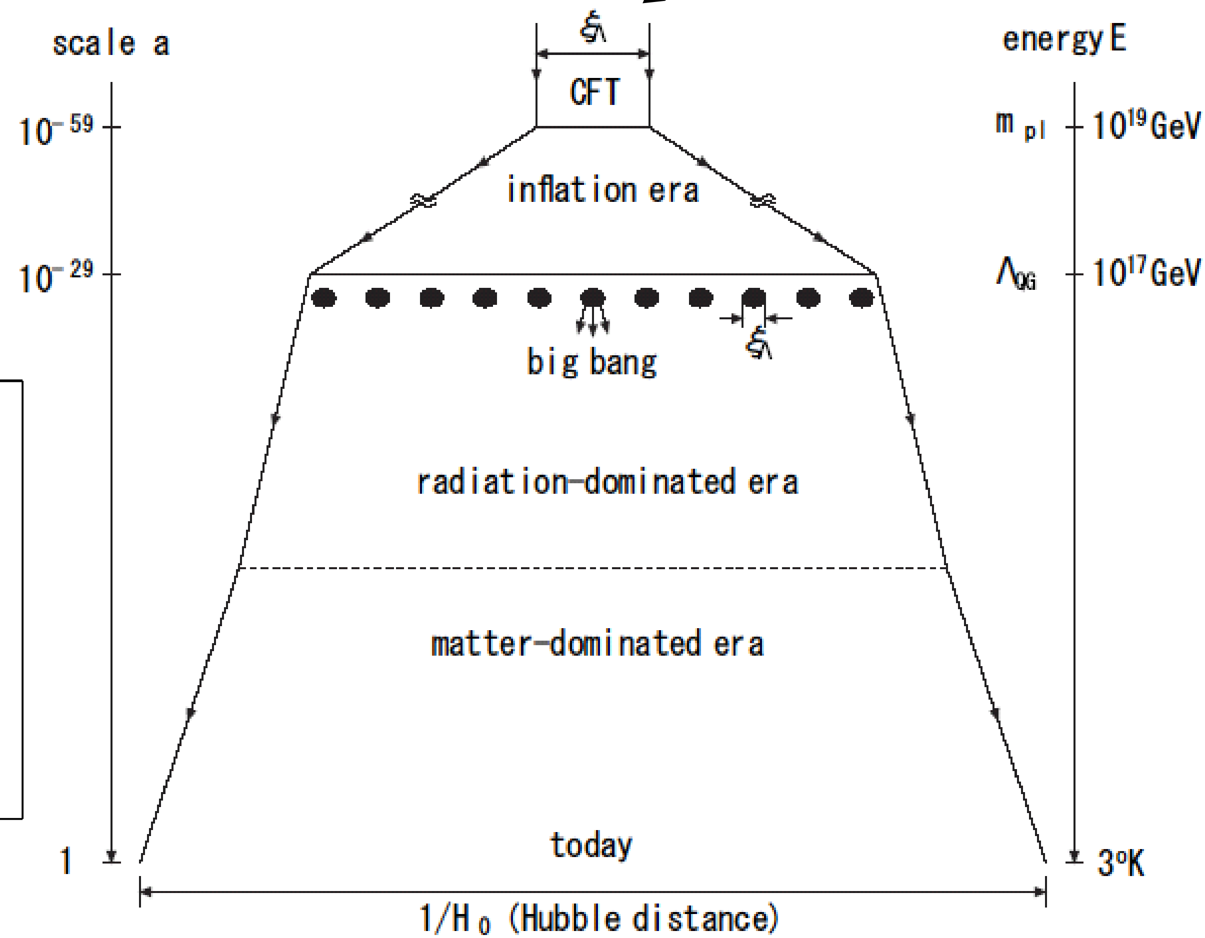
Expansion of the universe 10^{59}

Inflation era

$10^{30} \ (\Leftrightarrow \mathcal{N}_e = 70)$

Friedmann era

$10^{29} \ (\Leftrightarrow 10^{17} \mathrm{GeV}/2.7\mathrm{K})$



$1/H_0 \simeq 10^{59} \xi_\Lambda$ (~4000Mpc)

can be observed through CMB

Evolution of Scalar Fluctuations

WMAP observes quantum fluctuations of scalar curvature right before the spacetime phase transition

$$\left.\frac{\delta R}{R}\right|_{\tau_{\mathrm{P}}} \Longrightarrow \left.\frac{\delta R}{R}\right|_{\tau_\Lambda^-} \xrightarrow{\text{transition}} \left.\frac{\delta \rho}{\rho}\right|_{\tau_\Lambda^+} \Longrightarrow \left.\frac{\delta T}{T}\right|_{\text{today}}$$

fluctuations are getting small during inflation

spacetime sifts to classical universe

spectrum deformed by several dynamical scales

Remark:

The fluctuation we consider here expands rapidly enough to the size far from the horizon scale during inflation, and thus its spectrum does not disturbed by the dynamics near the transition

➔ we can directly see Planck scale spectrum

Estimation of Scalar Amplitude

Since scalar curvature has two derivatives, the amplitude of fluctuation near the transition point is to be the order of

$$\delta R \sim \Lambda_{\rm QG}^2$$

Dimensionless scalar fluctuation is thus estimated as

$$\sqrt{\left\langle \left(\frac{\delta R}{R} \right)^2 \right\rangle}\Bigg|_{\tau=\tau_\Lambda} \sim \frac{\Lambda_{\rm QG}^2}{12 H_{\rm D}^2} \simeq \frac{1}{12 \mathcal{N}_e^2} \sim 10^{-5}$$

(= root of amplidude)

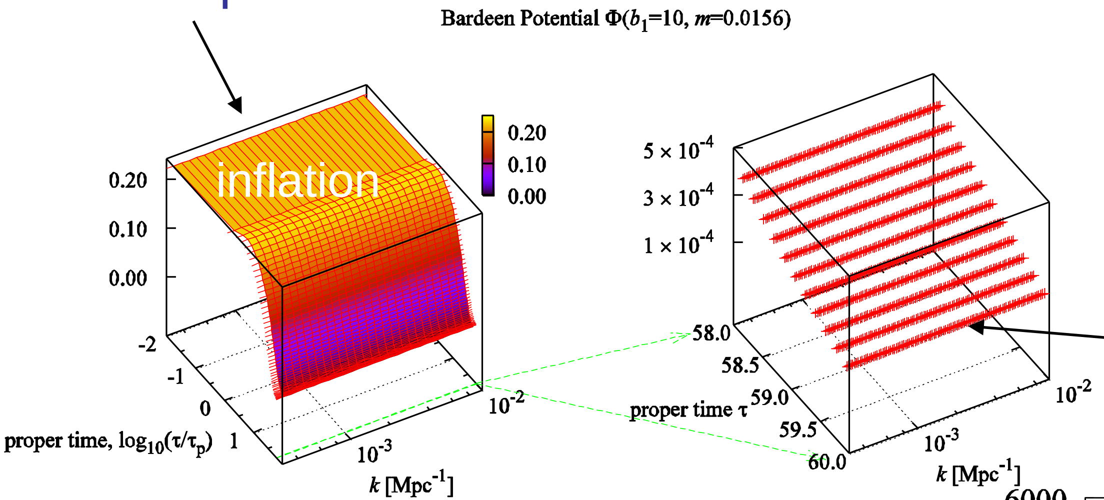
de Sitter curvature $R \simeq 12 H_{\rm D}^2$

This value is consistent with WMAP result

On the other hand, initial amplitude is given by 1/b ~ 10^{-2} for GUT
From these estimations, it seems that linear perturbation about inflationary solution become applicable

Evolution of Fluctuations from CFT to CMB

Scale-inv. spectrum at Planck time
with amp. = 1/b ~ 10^{-2}



From Planck length to
cosmological distance

$$10^{59} = 10^{30} + 10^{29}$$

inflation Friedmann

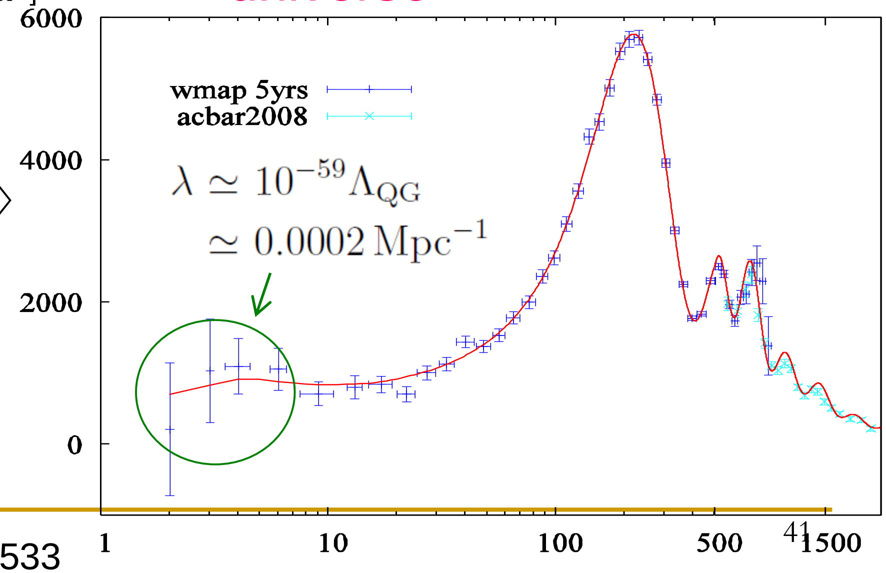
spectrum at transition point
= initial condition of Friedmann
universe

CMB spectrum is computed
using CMBFAST Fortran code

Initial condition is then set to
be almost scale invariant:

$$P_s(k) = A_s \left(\frac{k}{m}\right)^{n_s - 1 + v/\log(k^2/\lambda^2)}, \quad n_s = 1$$

~10^{-9} scale invariant



K.H., S.Horata and T. Yukawa, PRD 81(2010)083533

Summary

Basic Structure of Asymptotically Background Free Quantum Gravity

Renormalizable ABF quantum gravity does not have R^2 action

Conformal-factor dynamics is induced quantum mechanically

(when formulated using dim. reg., R^2 appears at order of D-4)

The kinetic term of conformal-factor field is given by

Riegert's Wess-Zumino action associated with conformal anomaly

BRST CFT + perturbation by the single coupling t

Beta function is negative

$$g_{\mu\nu} = \underline{e^{2\phi}}(\hat{g}_{\mu\nu} + \underline{t}h_{\mu\nu} + \cdots)$$

non-perturbative

The theory has background-metric independence in UV limit, which is represented as BRST conformal invariance

It implies that there is no particle picture beyond the Planck scale

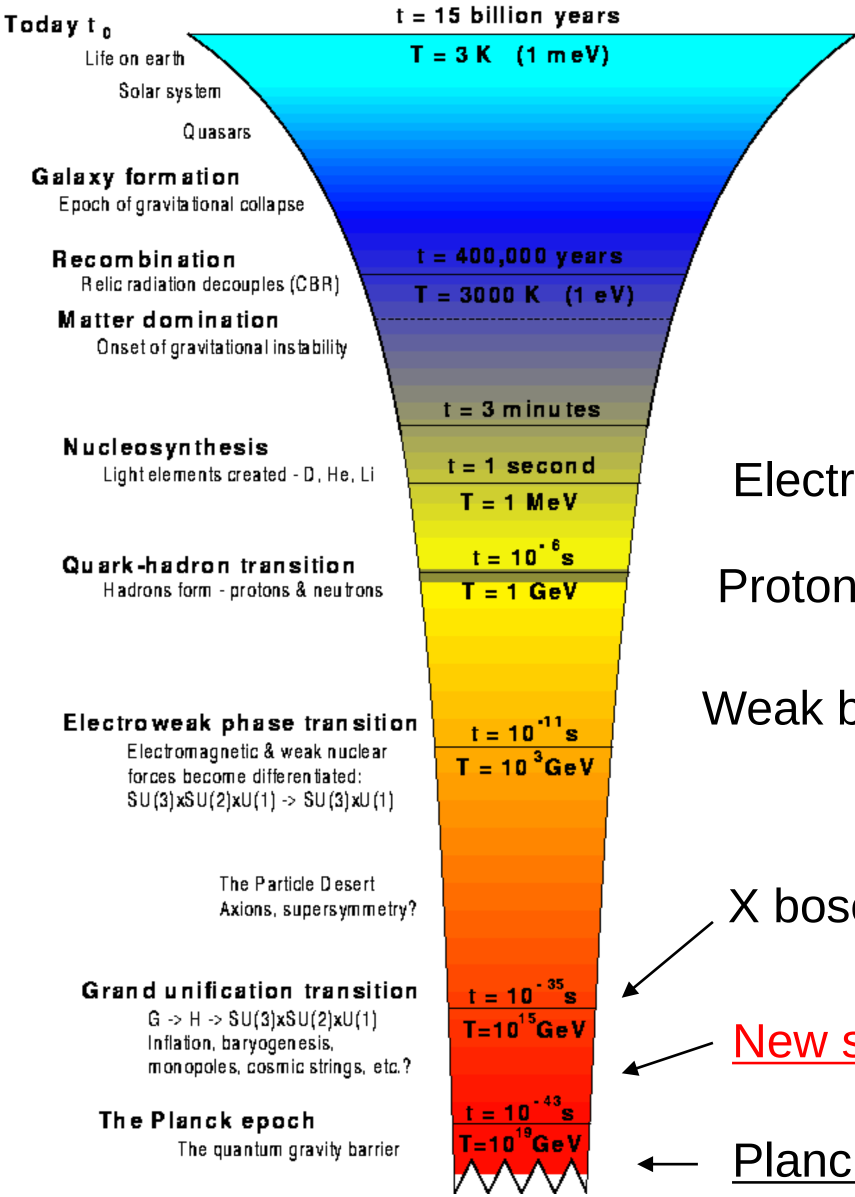
Physical Implications

- Repulsive effect in quantum gravity (the origin of expanding universe)
  - Inflation starts at Planck time and then fluctuations decrease
  - It also prevents black hole from collapsing to a point
- Asymptotically background free behavior
  (It indicates a realization of BRST CFT, not a free-particle theory, in UV limit)
  - Initially, scalar fluctuations dominate than tensor, as observed by WMAP
  - It predicts existence of novel dynamical scale

$$\Lambda_{\rm QG} \simeq 10^{17}\ {\rm GeV}$$

  - This scale divides classical and quantum spacetimes
    ➔ spacetime phase transition occurs
  - There is minimal length $\xi_\Lambda = 1/\Lambda_{\rm QG}$ we can measure
    (Spacetime is practically quantized even if without discretizing it)
- Quantum gravity spectrum
  - It is almost scale invariant due to conformal invariance
  - It can explain sharp fall-off of CMB low multipoles by $\Lambda_{\rm QG}$

Scales in the history of universe



Electron mass $m_e = 0.5\ \mathrm{MeV}$ (QED)

Proton mass $m_P = 1\ \mathrm{GeV}$ (QCD)

Weak boson mass $m_W = 80\ \mathrm{GeV}$ (EW theory)

X boson mass $m_X = 10^{16}\ \mathrm{GeV}$ (GUT)

New scale $\Lambda_{QG} \simeq 10^{17}\mathrm{GeV}$ (Quantum Gravity)

Planck mass $M_P = 10^{19}\ \mathrm{GeV}$

Appendix

Wess-Zumino Action and Euler Density

2D quantum gravity | 4D quantum gravity

modified

$$R \qquad \text{Euler conformal anomaly } \left(\delta_\phi \Gamma = \sqrt{-g}\,\Theta^\mu{}_\mu \right) \qquad E_4 = G_4 - \frac{2}{3}\nabla^2 R$$

relation

$$\sqrt{-g}R = \sqrt{-\bar{g}}\left(2\bar{\Delta}_2\phi + \bar{R}\right) \qquad \sqrt{-g}E_4 = \sqrt{-\bar{g}}\left(4\bar{\Delta}_4\phi + \bar{E}_4\right)$$

conf. inv. op.

$$\Delta_2 = -\nabla^2 \qquad \Delta_4 = \nabla^4 + 2R^{\mu\nu}\nabla_\mu\nabla_\nu - \frac{2}{3}R\nabla^2 + \frac{1}{3}\nabla^\mu R \nabla_\mu$$

Wess-Zumino action = local part of effective action Γ

$$-\frac{b_{\mathrm{L}}}{4\pi}\int d^2x \int_0^\phi d\phi \sqrt{-g}R = -\frac{b_{\mathrm{L}}}{4\pi}\int d^2x \sqrt{-\bar{g}}\left(\phi\bar{\Delta}_2\phi + \bar{R}\phi\right)$$

Liouville action

$$-\frac{b_1}{(4\pi)^2}\int d^4x \int_0^\phi d\phi \sqrt{-g}E_4 = -\frac{b_1}{(4\pi)^2}\int d^4x \sqrt{-\bar{g}}\left(2\phi\bar{\Delta}_4\phi + \bar{E}_4\phi\right)$$

Riegert action

Recently, from RG analysis of conf. anomaly using dim. reg., it has been shown that E_4 combination indeed arises. [K.H., PR D89(2014)104063]

Background-metric Independence

This QG model has background-metric indep. in UV limit ($t = 0$)

Outline of the proof

First, notice that the theory is invariant under a simultaneous shift: $\phi \to \phi - \omega$ and $\hat{g}_{\mu\nu} \to e^{2\omega}\hat{g}_{\mu\nu}$,
because it preserves the full metric field $g_{\mu\nu} = e^{2\phi}\hat{g}_{\mu\nu}$

Further, conformal-factor field is an integral variable in QG, and now it is treated exactly without introducing its own coupling

Thus, the measure is invariant under the shift $\phi' = \phi - \omega$ as

$$\int_{-\infty}^{\infty} [d\phi'] = \int_{-\infty}^{\infty} [d\phi]$$

Consequently, the theory becomes invariant under the conformal change such as

$$Z(e^{2\omega}\hat{g}) = Z(\hat{g})$$

Conformal Anomalies in Curved Space

From RG analysis, conformal anomalies (CA) associated with conformal couplings can be determined at all orders as

$$\theta^{\mu}{}_{\mu} = \frac{\beta_g}{4}[F^a_{\mu\nu}F^{a\mu\nu}] - \mu^{D-4}(\underline{\beta_a C^2_{\mu\nu\lambda\sigma} + \beta_b E_D})$$

where

$$E_D = G_D - 4\chi(D)\nabla^2 H$$

- GCA are unified into 2 forms only at all orders of perturbation
- CA are proportional to beta functions
- Familiar ambiguous $\nabla^2 R$ term is fixed completely

↑

known as trivial conformal anomaly

At D → 4 $\quad E_D \to E_4 = G_4 - \frac{2}{3}\nabla^2 R \quad$ proposed by Riegert in 1984

Comments On Conformal Anomaly

Conformal anomaly = conformal change of effective action:

$$\frac{\delta}{\delta\phi}\Gamma = \sqrt{-g}\,\Theta^{\mu}{}_{\mu}$$

violation of classical conf. inv.
(in curved space theory)

Effective action can be reconstructed by integrating it as

$$\Gamma = \int d^4x \int_0^{\phi} d\phi \sqrt{-g}\,\Theta^{\mu}{}_{\mu} + \underline{\text{nonlocal terms}}\ \text{indep. of}\ \phi$$

diff. inv.

Wess-Zumino action (local forms)

$$\phi^n F_{\mu\nu}^2,\ \phi^n \bar{C}_{\mu\nu\lambda\sigma}^2,\ \phi^{n+1}\bar{\Delta}_4\phi,\ \cdots$$

obtained by loop corrections

Physically, conformal anomaly is <u>not</u> an anomalous quantity
It arises to preserve diffeomorphism invariance

Furthermore, when going to <u>quantum gravity</u>, unlike in curved space theory, conformal anomalies play a significant role to <u>recover</u> conformal invariance, namely background-metric independence, as mentioned below

General Comments on Theoretical Structure of BRST Conformal Symmetry

First, the kinetic terms of both matter and gravitational fields must have "classical conformal invariance".

When only matter fields are quantized (= curved space theory), conformal invariance is always violated through Wess-Zumino actions associated with conformal anomalies.

However

When gravity is quantized further incorporating Wess-Zumino action properly, conformal invariance recovers exactly at the quantum level.

Thus, "conformal anomalies" are now necessary elements to preserve exact conformal invariance, namely diffeomorphism inv.

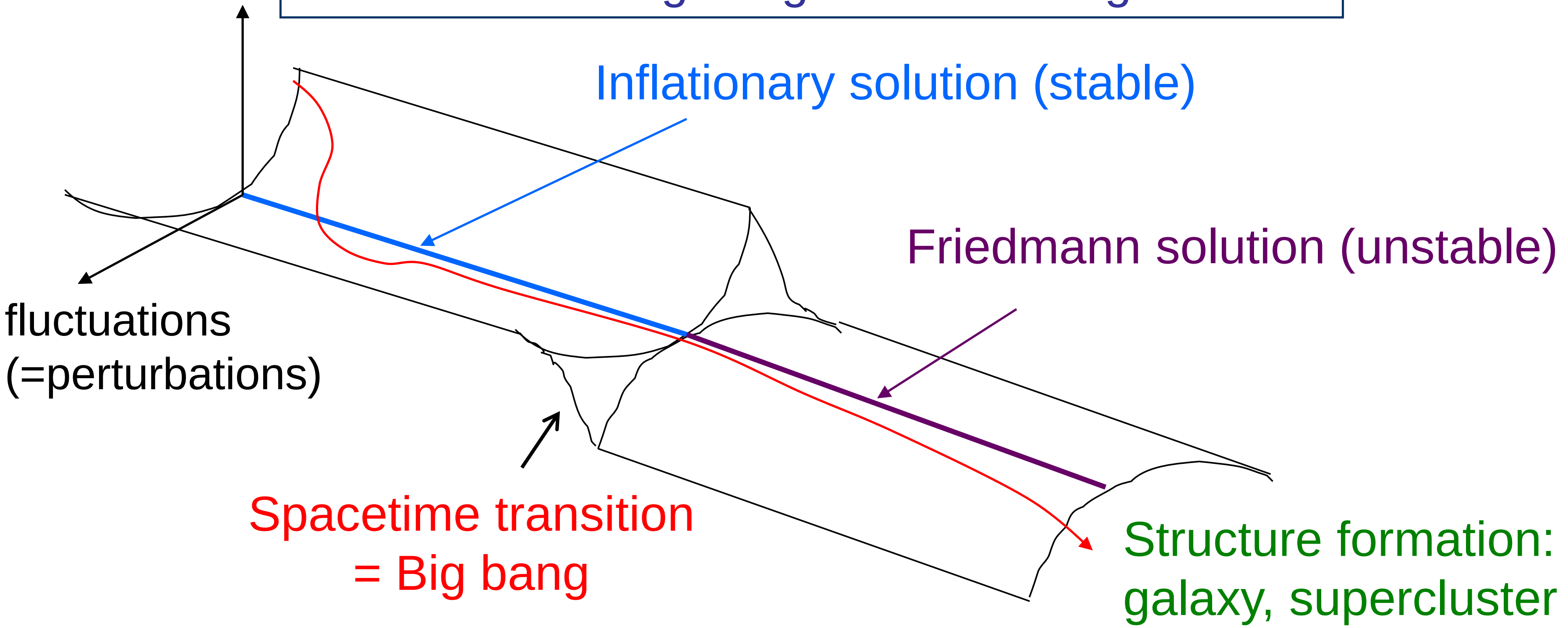
In order to construct the BRST operator at the quantum level, classical conformal invariance of the kinetic terms are necessary.

➔ This symmetry is known only in even dimensions, but not in odd

feedback

Sketch on Stability of Fluctuations

Fluctuations are getting smaller during inflation



In order that our universe continues about more than 10 billion years, fluctuations at big bang epoch must be quite small, because if not so our universe should have much deviated from Friedmann solution

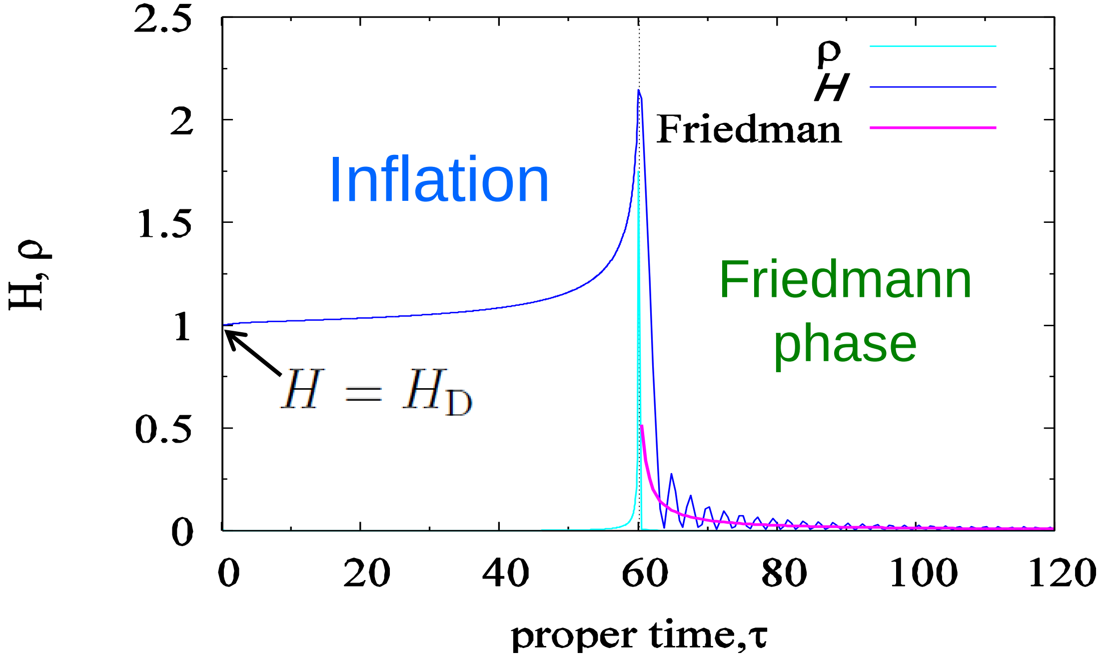
Energy Conservation and Big Bang

Energy conservation equation

matter density

$$\frac{b_1}{8\pi^2}B(\tau)\{2\partial_\eta^3\phi\partial_\eta\phi - \partial_\eta^2\phi\partial_\eta^2\phi\} - 3M_{\rm P}^2 e^{2\phi}\partial_\eta\phi\partial_\eta\phi + e^{4\phi}\rho = 0$$

Energy shift causes big bang



Inflationary solution indicates $\rho = 0$ initially

At $\tau_\Lambda = 1/\Lambda_{\rm QG}$, dynamical factor vanishes and then gravitational energy shifts to matter density ρ

⇧

Interactions that create matter density is given by Wess-Zumino actions like $\phi F_{\mu\nu}^2$

Linear Fluctuation Variables

In the following, we compute evolution eqs of linear fluctuations from Planck time to spacetime transition time

(The validity of approximation will be discussed after solving eqs)

Scalar perturbations (= Bardeen's gravitational potentials)

$$ds^2 = a^2[-(1+2\Psi)d\eta^2 + (1+2\Phi)d\mathbf{x}^2]$$

↖ inflationary background

Initially $(t_r = 0)$ $\Phi = \Psi$ (= conformal-factor perturbation)

At $t_r \neq 0$ $\Phi \neq \Psi$

Tensor perturbations (not discussed here)

Due to asymptotically free behavior, tensor fluctuations will be quite small initially ➔ tensor-scalar ratio is small

Coupled Dynamical Evolution Equations

Equations is derived from Riegert + Weyl + Einstein + Matter system

Here, consider two combinations independent of matter sector

First equation (=trace of EM tensor)

$$\left(\partial\!\!\!/^2 = \partial_i \partial^i \right.$$

3D Laplacian

$$\frac{b_1}{8\pi^2} B(\tau) \Big\{ -2\partial_\eta^4 \Phi - 2\partial_\eta \phi \partial_\eta^3 \Phi + \left(-8\partial_\eta^2 \phi + \frac{10}{3} \partial\!\!\!/^2 \right) \partial_\eta^2 \Phi$$
$$+ \left(-12\partial_\eta^3 \phi + \frac{10}{3} \partial_\eta \phi \partial\!\!\!/^2 \right) \partial_\eta \Phi + \left(\frac{16}{3} \partial_\eta^2 \phi - \frac{4}{3} \partial\!\!\!/^2 \right) \partial\!\!\!/^2 \Phi$$
$$+2\partial_\eta \phi \partial_\eta^3 \Psi + \left(8\partial_\eta^2 \phi + \frac{2}{3} \partial\!\!\!/^2 \right) \partial_\eta^2 \Psi + \left(12\partial_\eta^3 \phi - \frac{10}{3} \partial_\eta \phi \partial\!\!\!/^2 \right) \partial_\eta \Psi$$
$$+ \left(-\frac{16}{3} \partial_\eta^2 \phi - \frac{2}{3} \partial\!\!\!/^2 \right) \partial\!\!\!/^2 \Psi \Big\}$$

Riegert term

$$+M_{\mathrm{P}}^2 e^{2\phi} \Big\{ 6\partial_\eta^2 \Phi + 18\partial_\eta \phi \partial_\eta \Phi - 4\partial\!\!\!/^2 \Phi - 6\partial_\eta \phi \partial_\eta \Psi$$
$$+ \left(12\partial_\eta^2 \phi + 12\partial_\eta \phi \partial_\eta \phi - 2\partial\!\!\!/^2 \right) \Psi \Big\} = 0$$

Einstein term

(there is no contribution from Weyl term in this equation)

Second Equation (=constraint equation)

reduces to 2nd order by factoring out $\bar{\partial}^2$

$$\frac{b_1}{8\pi^2}B(\tau)\Big\{\frac{4}{3}\partial_\eta^2\Phi + 4\partial_\eta\phi\partial_\eta\Phi + \Big(\frac{28}{3}\partial_\eta^2\phi - \frac{8}{3}\partial_\eta\phi\partial_\eta\phi - \frac{8}{9}\bar{\partial}^2\Big)\Phi$$

$$-\frac{4}{3}\partial_\eta\phi\partial_\eta\Psi + \Big(-\frac{4}{3}\partial_\eta^2\phi + \frac{8}{3}\partial_\eta\phi\partial_\eta\phi - \frac{4}{9}\bar{\partial}^2\Big)\Psi\Big\}$$

Riegert term

$$+\frac{2}{t_r^2(\tau)}\Big\{4\partial_\eta^2\Phi - \frac{4}{3}\bar{\partial}^2\Phi - 4\partial_\eta^2\Psi + \frac{4}{3}\bar{\partial}^2\Psi\Big\}$$

Weyl term

$$-2M_P^2 e^{2\phi}\{\Phi + \Psi\} = 0$$

Einstein term

$\Rightarrow$

Initially $(t_r = 0)$ $\quad \Phi = \Psi \quad$ because Weyl tensor vanishes (← asymptotic freedom)

Finally $(t_r = \infty)$ $\quad \Phi = -\Psi \quad$ because Einstein term dominates

↑

conformal dynamics disappears : B, $1/tr^2$ → 0

Summary of Quantum Gravity Dynamics

K.H, PRD85(2012)024028; PRD86(2012)124006

- BRST conformal symmetry arises in UV limit ($t \rightarrow 0$ limit)
  = all spacetimes connecting each other under conformal transformations
  become gauge-equivalent: $g_{\mu\nu} \approx \Omega g_{\mu\nu}$
  = a representation of background-metric independence

Notice:
not a free theory
(a certain CFT)

(like BRST Virasoro sym. in 2DQG)

We can go beyond the Planck scale !!

guaranteed by asymptotic-free behavior of the coupling t

- Physical (BRST inv.) states are "primary scalars" only
  ➔ Primordial spectrum is a scale-invariant and scalar-like
  (tensor is small of order t^2 in UV limit)

- Predicts stable inflation of Starobinsky-type
  It starts about Planck scale and ends at the IR scale $\Lambda_{QG} \simeq 10^{17}\mathrm{Gev}$
  (This scale also explains the sharp-falloff at low multipoles of CMB)

K.H., S. Horata and T.Yukawa, PRD74(2006)123502; PRD81(2010)083533

Initial Conditions at Planck Time

Initial condition = two-point function of conformal-factor field

$$\langle \varphi(\tau_i, \mathbf{x}) \varphi(\tau_i, \mathbf{x}') \rangle = -\frac{1}{4b_1} \log\left(m^2 |\mathbf{x} - \mathbf{x}'|^2 \right)$$

$\tau_i = 1/E_i$

$(E_i \geq H_{\mathbf{D}})$

$\Phi = \Psi$ mode

In momentum space

$$-\log\left(m^2 |\mathbf{x}|^2 \right) = \int_{k>\epsilon} \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{4\pi^2}{k^3} e^{i\mathbf{k}\cdot\mathbf{x}} \cancel{- \log\left(\frac{m^2}{\epsilon^2 e^{2\gamma - 2}} \right)}$$

$$\left(\begin{aligned} m &= a(\tau_i) H_{\mathbf{D}} \\ k &= a(\tau_i) p \end{aligned} \right)$$

delta func.
in Fourier sp.

comoving

physical

We obtain scale-invariant scalar spectrum

$$P(\tau_i, k) = \frac{k^3}{2\pi^2} \langle |\tilde{\varphi}(\tau_i, \mathbf{k})|^2 \rangle = \frac{1}{2b_1}$$

$b_1 \simeq 10$

for GUT models

Harrison-Zel'dovich spectrum

positive-definite = physical

← positivity of Riegert action